New Rating System – worked example

Introduction

Given the recent changes to the new rating system in response to the system's application to the World Seniors tournaments, I thought it might interest some people to provide a worked example of exactly how new ratings are worked out using the new system as it is now.
To do this, I'll be 'rating' a dummy tournament between 4 players, A, B, C, and D, where A has an established rating of 1550, B has a provisional rating of 1500 (and has previously played 7 tournament games), C has an established rating of 1450, and D is a new player.
In this small tournament, the players have each played 5 games – A and C have played each other once, and each of the other twice, and B and D have played each other once. A has gained 2 wins out of 5, B has 0 out of 5, C has 5 out of 5, and D has 3 out of 5.

First pass - Provisional ratings

Players who start the tournament with a provisional rating (having played fewer than 30 NZ tournament games) all have their new ratings calculated using the provisional formulas before the new ratings for any players with established (30 or more NZ tournament games) ratings are calculated. If the number of games in the current tournament takes a provisionally-rated player up to 30 or more games, their new rating will be treated as an established rating in their next tournament.
A provisional rating is calculated by taking the average rating of the player's opponents' ratings and applying a ratings gain or loss to it. The ratings gain is calculated from the players win rate using this formula:

-log(1/winrate - 1)*313

(where log is the natural (base e) logarithm – LN(x) in Excel)

Given that a win rate of 0 (e.g. 0 wins out of 5 games) would give a division-by-zero error in this formula, and a win rate of 1 (e.g. 5 wins out of 5 games) would give an infinite rating gain, the actual win rate is scaled using this formula:

((wins-games/2)*(games-2)/games+games/2)/games

This formula scales the winrate to between one game out of the total number of games and one less than the total number of games. E.g. 0 wins out of 5 becomes 1 win out of 5 (or a win rate of 0.2 rather than 0), 2.5 wins out of 5 stays as is, and 3 wins out of 5 becomes 2.8 wins out of 5 (or a win rate of 0.56 rather than 0.6).
Given these formulas, the initial provisional ratings would be worked out as follows:

For B: the win rate of 0 becomes 0.2 under the second formula, then, under the first formula:1/0.2 = 5
-> 5-1 = 4
-> log(4) = 1.386294361
-> 1.386294361 *313 = 433.910135
-> 433.910135 * -1 = -433.910135

The average (mean) of the opponents' ratings is (twice A's rating + twice C's rating + D's rating) / 5 games = (1550 + 1550 + 1450 + 1450 + 0) / 5 = 6000/5 = 1200
Adding the ratings loss gives 1200 – 433.910135, for an initial provisional rating of 766.090

For D: the same calculations go as follows:
Raw win rate: 3 out of 5 = 0.6, scaled to 0.56
Then: 1/0.56 = 1.785714286
-> 1.785714286 – 1 = 0.785714286
-> log(0.785714286) = -0.241162056
-> -0.241162056 * 313 = -75.48372367
-> -75.48372367 * -1 = 75.48372367

(1550 + 1550 + 1450 + 1450 + 1500)/5 = 1500
1500 + 75.48372367 = 1575.484

We then replace B's start rating of 1500 with 766.090, and replace D's start rating of 0 with 1575.484, and rerun the calculations until these initial provisional ratings stop changing significantly.

name	rating	status	games	wins	win rate	scaled	diff	prov1
A	1550		5	2	0.4			
B (7)	1500	(prov)	5	0	0	0.2	-433.910	766.090
C	1450		5	5	1			
D (0)	0	(new)	5	3	0.6	0.56	75.484	1575.484

name	prov2	prov3	prov4	prov5	prov6	prov7	prov8
A							
B (7)	1081.187	1051.830	1064.434	1063.260	1063.764	1063.717	1063.737
C							
D (0)	1428.702	1491.721	1485.850	1488.371	1488.136	1488.237	1488.227

At this stage, the latest provisional rating is the final provisional rating for new players, while it is averaged out with the pre-tournament provisional rating for provisionally-rated players, according to the number of games played before the tournament and during the tournament. (In the next revision of the rating system, this averaging step is likely to be deferred until after the 'second pass' below.)
So, for B: (1500 * 7 + 1063.737 * 5) is divided by 12 (being 7 games before plus 5 games during) giving 1318.224, while D remains on 1488.227.

Second pass - Established ratings

Now that the provisional ratings have been worked out for new and provisionally-rated players, they can be used to work out the new ratings for players with established ratings.
This is done by summing the expected winrate against each opponent (worked out from the difference between the two player's starting or provisional ratings) to give an expected number of wins out of all the games played. A fixed number of points (the k-factor, which you can think of as the importance of the current games being rated to the player's overall rating) is added or subtracted according to the difference between the player's wins and their expectancy.
The formula to work out the expected win rate for a single game is:

1 / (1 + Exp((player_rating - opponent_rating) / -313))

The Exp() function here gives e to the power of its argument.

So, to work out A's expected win rate against C for a single game: 1550 – 1450 = 100
-> 100 / -313 = -0.319488818
-> exp(-0.319488818) = 0.726520326
-> 1 + 0.726520326 = 1.726520326
-> 1 / 1.726520326 = 0.579199668

To work out C's expected win rate against A for a single game: 1450 – 1550 = -100
-> -100 / -313 = 0.319488818
-> exp(0.319488818) = 1.376423981
-> 1 + 1.376423981 = 2.376423981
-> 1 / 2.376423981 = 0.420800332

You may notice that 0.579199668 + 0.420800332 = 1. If A and C were to play each other 100 times (without their ratings changing during that time), A would be expected to win almost 58 games, and C would be expected to win just over 42 games. This is the same in the new system for any two players whose ratings are 100 points apart.

A's expectancy against B is 0.632542918, and against D is 0.549179832, so, as A plays B and D twice each, and C once, A's total expectancy is 0.632542918 + 0.632542918 + 0.579199668 + 0.549179832 + 0.549179832 = 2.942645168 out of 5 games.

C's expectancy against B is 0.603724947, and against D is 0.469505049, so, as C plays B and D twice each, and A once, C's total expectancy is 0.420800332 + 0.603724947 + 0.603724947 + 0.469505049 + 0.469505049 = 2.567260324 out of 5 games.

The formula for the k-factor is: ((3000 - rating) / 1000) * gamesplayed

So, A's k-factor = ((3000 – 1550) / 1000) * 5 = 7.25
C's k-factor = ((3000 – 1450) / 1000) * 5 = 7.75

The rating change is then given by multiplying the difference between the player's wins and their expectancy by their k-factor.

For A: (2 - 2.942645168) * 7.25 = -6.834177471
For C: (5 - 2.567260324) * 7.75 = 18.85373249

Final steps

There are only two other matters to consider.

First, did any of the established players earn accelerator points?
Under the current scenario, no. The cutoff for earning accelerator points is to win more than 3 1/3 games more than your expectancy. Here, C wins 2.432739676 games more than their expectancy – not quite enough.

If, however, C had started with an established rating of 450 rather than 1450, and everything else remained the same, B and D's provisional ratings would work out as 1109.891 and 988.228, A and C's expectancies out of 5 would become 4.292422821 and 0.549346253, the k-factors would remain the same, and A and C's rating changes would become -16.62006545 and 56.74583528.

Under this scenario, C's 5 wins would exceed their 0.549346253 expectancy by 4.450653747 – well above 3 1/3 wins. Accelerator points are calculated by doubling the points in excess of the cutoff. If C had exceeded their expectancy by exactly 3 1/3 games, their rating change would have been 42.5, so the accelerator points are 56.74583528 - 42.5 = 14.24583528.

Feedback points are then awarded at a rate of 1/20 of the accelerator points for each game played against the player who gained the accelerator points. Provisional players are not eligible for accelerator points (as they do not have an expectancy defined) but will still receive feedback points for each game played against a recipient of accelerator points. In this case, A would receive 0.712291764 points for having played C once, and B and D will each receive twice this (1.424583528) as they each played C twice.

Second, participation points are added at a rate of 1 per 3 games played. All players here played 5 games, so they each each 5/3 = 1 2/3 participation points. The rating change, accelerator and feedback points, and participation points are then added to the final provisional rating, which is rounded to the nearest integer, and any sub-zero ratings are brought up to zero.

name	prov	expected	k-factor	change	accel/fb	pps	total
A	1550	4.292422821	7.25	-16.62006545	0.712291764	1 2/3	1536
B (7)	1109.891				1.424583528	1 2/3	1113
C	450	0.549346253	12.75	56.74583528	14.24583528	1 2/3	523
D (0)	988.228				1.424583528	1 2/3	991